A Verified Lisp Implementation for A Verified Theorem Prover

Scheme workshop 2016, Nara, Japan

Magnus O. Myreen — University of Cambridge, but now at Chalmers University of Technology Jared Davis — Centaur Technology, Inc., but now at Apple

Result:

A Verified Lisp Implementation for A Verified Theorem Prover

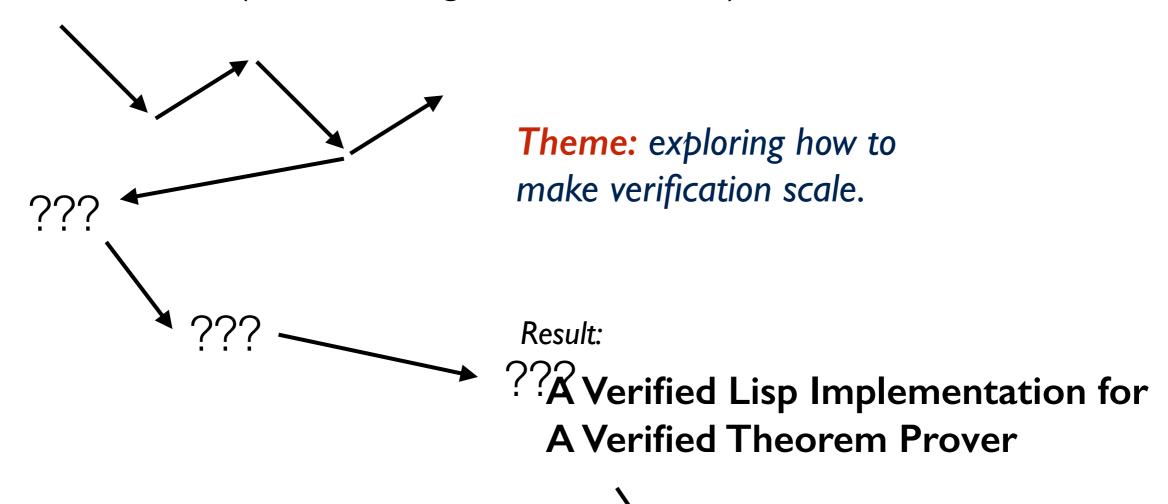
Claim:

The most comprehensive proof-based evidence of a theorem prover's soundness to date.

This talk: The Journey

2005:

I'm a PhD student working on verification of machine code (factorial, length of a linked list)



The start:

I'm a PhD student working on verification of machine code (factorial, length of a linked list)

Context: interactive theorem proving

Aim: to prove deep functional properties of machine code.

Proofs are performed in HOL4 — a fully expansive theorem prover

HOL4 theorem prover

HOL4 kernel

All proofs expand at runtime into primitive inferences in the HOL4 kernel.

The kernel implements the axioms and inference rules of higher-order logic.

Context: interactive theorem proving



photo idea: Larry Paulsson

Machine code

Machine code,

E1510002 B0422001 C0411002 01AFFFFB

is impossible to read, write or maintain manually.

However, for theorem-prover-based formal verification:

machine code is clean and tractable!

Reason:

- ▶ all types are concrete: word32, word8, bool.
- state consists of a few simple components: a few registers, a memory and some status bits.
- each instruction performs only small well-defined updates.

Challenges of Machine Code

machine code

code

ARM/x86/PowerPC model (1000...10,000 lines each)

correctness

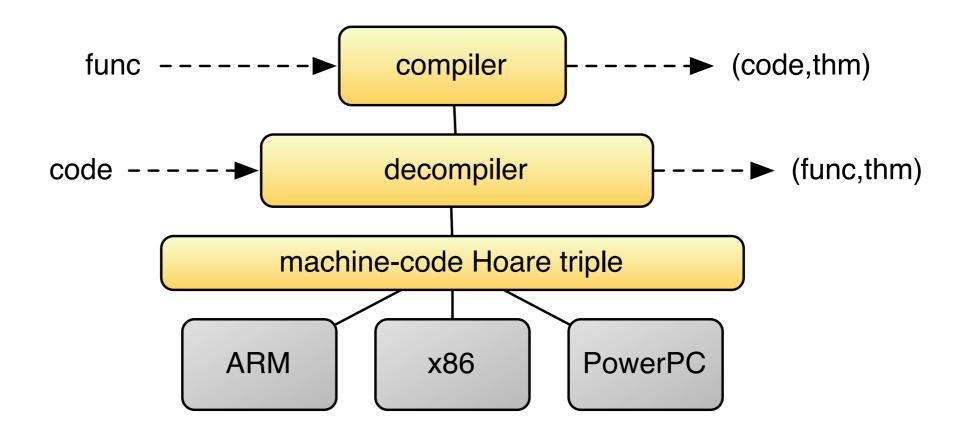
{P} code {Q}

Challenges:

- several large, detailed models
- unstructured code
- very low-level and limited resources

Infrastructure

During my PhD, I developed the following infrastructure:



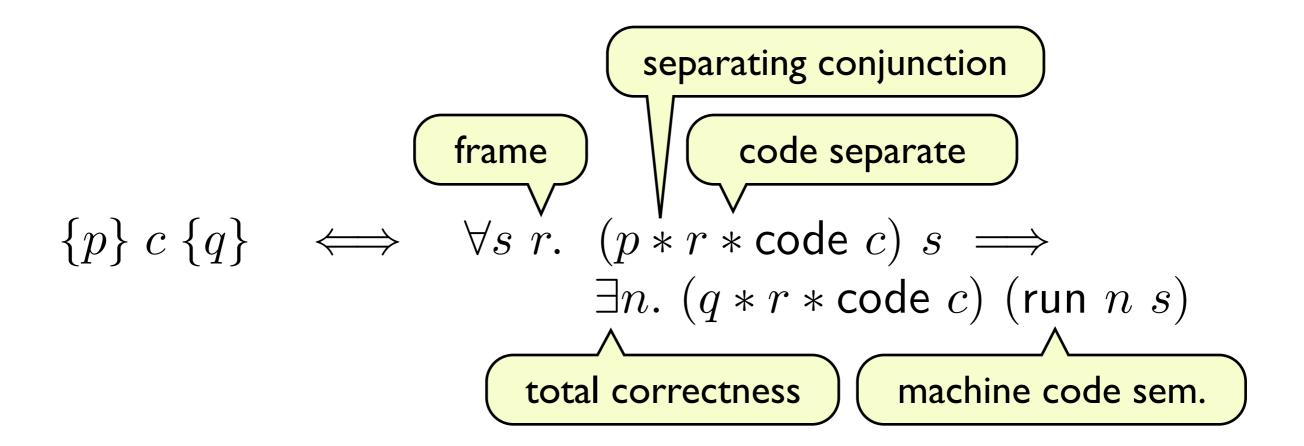
... each part will be explained in the next slides.

Hoare triples

Each model can be evaluated, e.g. ARM instruction add r0,r0,r0 is described by theorem:

As a total-correctness machine-code Hoare triple:

Definition of Hoare triple



Program logic can be used directly for verification. But direct reasoning in this embedded logic is tiresome.

Decompiler

Decompiler automates Hoare triple reasoning.

Example: Given some ARM machine code,

```
0: E3A00000 mov r0, #0
4: E3510000 L: cmp r1, #0
8: 12800001 addne r0, r0, #1
12: 15911000 ldrne r1, [r1]
16: 1AFFFFFB bne L
```

the decompiler automatically extracts a readable function:

$$f(r_0, r_1, m) = \text{let } r_0 = 0 \text{ in } g(r_0, r_1, m)$$
 $g(r_0, r_1, m) = \text{if } r_1 = 0 \text{ then } (r_0, r_1, m) \text{ else }$
 $\text{let } r_0 = r_0 + 1 \text{ in }$
 $\text{let } r_1 = m(r_1) \text{ in }$
 $g(r_0, r_1, m)$

Decompilation, correct?

Decompiler automatically proves a certificate theorem:

```
f_{pre}(r_0, r_1, m) \Rightarrow
{ (R0, R1, M) is (r_0, r_1, m) * PC p * S}
p : E3A00000 E3510000 12800001 15911000 1AFFFFB
{ (R0, R1, M) is f(r_0, r_1, m) * PC (p + 20) * S}
```

which informally reads:

for any initially value (r_0, r_1, m) in reg 0, reg 1 and memory, the code terminates with $f(r_0, r_1, m)$ in reg 0, reg 1 and memory.

Decompilation verification example

To verify code: prove properties of function f,

$$\forall x \mid a \mid m. \mid list(I, a, m) \Rightarrow f(x, a, m) = (length(I), 0, m)$$

 $\forall x \mid a \mid m. \mid list(I, a, m) \Rightarrow f_{pre}(x, a, m)$

since properties of f carry over to machine code via the certificate.

Proof reuse: Given similar x86 and PowerPC code:

31C085F67405408B36EBF7

38A000002C140000408200107E80A02E38A500014BFFFFF0

which decompiles into f' and f'', respectively. Manual proofs above can be reused if f = f' = f''.

Decompilation how to

```
{ R0 i * R1 j * PC p }
p+0:
{ R0 (i+j) * R1 j * PC (p+4) }
{ R0 i * PC (p+4) }
\{ R0 (i >> I) * PC (p+8) \}
{ LR Ir * PC (p+8) }
p+8:
{ LR Ir * PC Ir }
{ R0 i * R1 j * LR lr * PC p }
{ R0 ((i+j)>>1) * R1 j * LR lr * PC lr }
```

How to decompile:

```
e08100000
          add
                r0, r1, r0
                r0, r0, #1
          lsr
e1a0000000
e12fffffe
                lr
          bx
```

- I. derive Hoare triple theorems using Cambridge ARM model
- 2. compose Hoare triples
- 3. extract function

(Loops result in recursive functions.)

p:e0810000 e1a000a0 e12fff1e

$$3 \longrightarrow avg(i,j) = (i+j) >> 1$$

Decompiler cont.

Implementation:

- ▶ ML program which fully automatically performs forward proof
- no heuristics and no dangling proof obligations
- loops result in tail-recursive functions

Case studies:

- verified copying garbage collector
- bignum library routines

Part 2:

I want more automation and abstraction!

Proof-producing compilation

Synthesis often more practical. Given function f,

$$f(r_1) = \text{if } r_1 < 10 \text{ then } r_1 \text{ else let } r_1 = r_1 - 10 \text{ in } f(r_1)$$

our *compiler* generates ARM machine code:

```
E351000A L: cmp r1,#10
2241100A subcs r1,r1,#10
2AFFFFC bcs L
```

and automatically proves a certificate HOL theorem:

```
\vdash \{ R1 \ r_1 * PC \ p * s \}
p : E351000A \ 2241100A \ 2AFFFFFC
\{ R1 \ f(r_1) * PC \ (p+12) * s \}
```

Compilation, example cont.

One can prove properties of f since it lives inside HOL:

$$\vdash \ \forall x. \ f(x) = x \bmod 10$$

Properties proved of f translate to properties of the machine code:

```
\vdash \{R1 \ r_1 * PC \ p * s\}
p : E351000A \ 2241100A \ 2AFFFFFC
\{R1 \ (r_1 \ mod \ 10) * PC \ (p+12) * s\}
```

Additional feature: the compiler can use the above theorem to extend its input language with: let $r_1 = r_1 \mod 10$ in _

Implementation

To compile function *f*:

- 1. generate, without proof, code from input *f*;
- 2. decompile, with proof, a function f' from generated code;
- 3. prove f = f'.

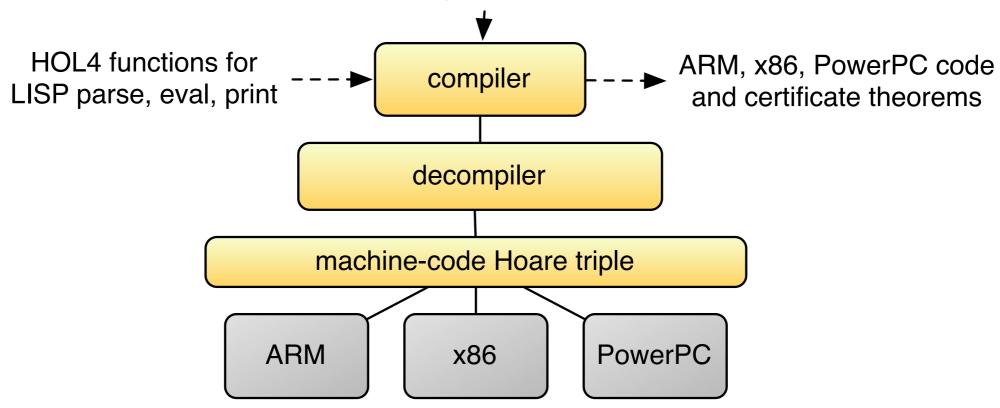
Features:

- code generation completely separate from proof
- supports many light-weight optimisations without any additional proof burden: instruction reordering, conditional execution, dead-code elimination, duplicate-tail elimination, ...
- allows for significant user-defined extensions

Infrastructure again

Idea: create LISP implementations via compilation.

verified code for LISP primitives car, cdr, cons, etc.



Lisp formalised

LISP s-expressions defined as data-type SExp:

Num : $\mathbb{N} \to \mathsf{SExp}$

 $Sym : string \rightarrow SExp$

 $\mathsf{Dot} \; : \; \mathsf{SExp} \to \mathsf{SExp} \to \mathsf{SExp}$

LISP primitives were defined, e.g.

$$cons x y = Dot x y$$

$$car (Dot x y) = x$$

$$plus (Num m) (Num n) = Num (m + n)$$

The semantics of LISP evaluation was taken to be Gordon's formalisation of 'LISP 1.5'-like evaluation

Extending the compiler

We define heap assertion 'lisp $(v_1, v_2, v_3, v_4, v_5, v_6, I)$ ' and prove implementations for primitive operations, e.g.

```
is_pair v_1 \Rightarrow { lisp (v_1, v_2, v_3, v_4, v_5, v_6, I) * pc p } p : E5934000 { lisp <math>(v_1, car \ v_1, v_3, v_4, v_5, v_6, I) * pc (p + 4) } size v_1 + size \ v_2 + size \ v_3 + size \ v_4 + size \ v_5 + size \ v_6 < I \Rightarrow { lisp (v_1, v_2, v_3, v_4, v_5, v_6, I) * pc p } p : E50A3018 E50A4014 E50A5010 E50A600C ... { lisp <math>(cons \ v_1 \ v_2, v_2, v_3, v_4, v_5, v_6, I) * pc (p + 332) }
```

with these the compiler understands:

```
let v_2 = \operatorname{car} v_1 in ...
let v_1 = \operatorname{cons} v_1 \ v_2 in ...
```

Reminder

```
{ R0 i * R1 j * PC p }
p+0: e0810000
{ R0 (i+j) * R1 j * PC (p+4) }
{ R0 i * PC (p+4) }
p+4: e1a000a0
\{ R0 (i >> I) * PC (p+8) \}
{ LR Ir * PC (p+8) }
p+8: e12fff1e
{ LR Ir * PC Ir }
{ R0 i * R1 j * LR lr * PC p }
p:e0810000 e1a000a0 e12fff1e
```

{ R0 ((i+j)>>1) * R1 j * LR lr * PC lr }

How to decompile:

We change these triples to be about lisp heap. Result: more abstraction.

eizttie bx li

- I. derive Hoare triple theorems using Cambridge ARM model
- 2. compose Hoare triples
- 3. extract function

(Loops result in recursive functions.)

 $3 \longrightarrow avg(i,j) = (i+j) >> 1$

2

The final case study of my PhD

TPHOLS'09

Verified LISP implementations on ARM, x86 and PowerPC

Magnus O. Myreen and Michael J. C. Gordon

Computer Laboratory, University of Cambridge, UK

Abstract. This paper reports on a case study, which we believe is the first to produce a formally verified end-to-end implementation of a functional programming language running on commercial processors. Intertional programming language running on commercial processors. Intertional programming language running on commercial processors. ARM, tional programming language running on commercial processors. Intertional programming language running on commercial processors. ARM, and proved to correctly parse, evaluate preters for the core of McCarthy's LISP 1.5 were implemented in ARM, and proved to correctly parse, evaluate and print LISP s-expressions. The proof of evaluation required working and print LISP s-expressions. The proof of memory allocation and garbage on top of verified implementations of memory allocation and proved to correctly parse.

Running the Lisp interpreter







Nintendo DS lite (ARM) MacBook (x86) old MacMini (PowerPC)

```
(pascal-triangle '((1)) '6)
```

returns:

```
((1 6 15 20 15 6 1)
 (1 5 10 10 5 1)
 (1 \ 4 \ 6 \ 4 \ 1)
 (1 \ 3 \ 3 \ 1)
 (1 \ 2 \ 1)
 (1 \ 1)
 (1))
```

Part 3:

A sudden need for a serious Lisp implementation.

Two projects meet

My theorem prover is written in Lisp. Can I try your verified Lisp?

Umm.. sure!

Does your Lisp support ..., ... and ...?

No, but it could ...

Jared Davis

Magnus Myreen

A self-verifying theorem prover

Verified Lisp implementations



verified LISP on ARM, x86, PowerPC

Running Milawa





verified LISP on ARM, x86, PowerPC

Milawa's bootstrap proof:

- 4 gigabyte proof file:>500 million unique conses
- takes 16 hours to run on a state-of-the-art runtime (CCL)



Running Milawa





Jitawa: verified LISP with JIT compiler

Milawa's bootstrap proof:

- 4 gigabyte proof file:>500 million unique conses
- takes 16 hours to run on a state-of-the-art runtime (CCL)

Result:

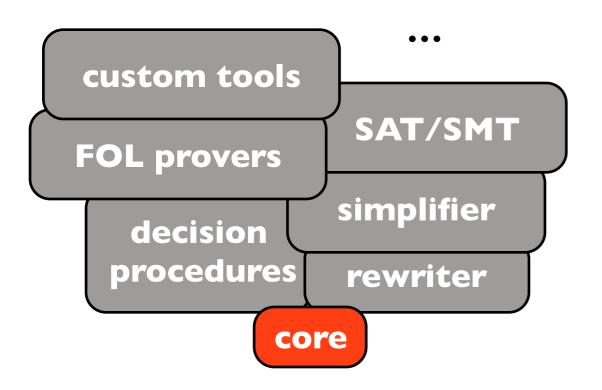
a new verified Lisp which is able to host the Milawa thm prover

A short introdution to



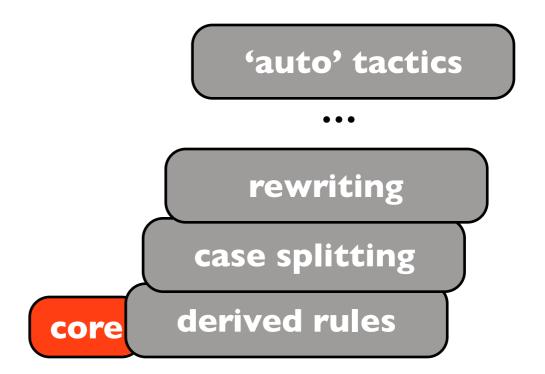
- Milawa is styled after theorem provers such as NQTHM and ACL2,
- has a small trusted logical kernel similar to LCF-style provers,
- ... but does not suffer the performance hit of LCF's fully expansive approach.

Comparison with LCF approach



LCF-style approach

- all proofs pass through the core's primitive inferences
- extensions steer the core



the Milawa approach

- all proofs must pass the core
- the core proof checker can be replaced at runtime

Requirements on runtime

Milawa uses a subset of Common Lisp which

is for most part first-order pure functions over natural numbers, symbols and conses,

uses primitives: cons car cdr consp natp symbolp

equal + - < symbol - < if

macros: or and list let let* cond

first second third fourth fifth

and a simple form of lambda-applications.

(Lisp subset defined on later slide.)

Requirements on runtime

...but Milawa also

- uses destructive updates, hash tables
- prints status messages, timing data
- uses Common Lisp's checkpoints
- forces function compilation
- makes dynamic function calls
- can produce runtime errors

not necessary

runtime must support

(Lisp subset defined on later slide.)

Runtime must scale

Designed to scale:

- just-in-time compilation for speed
 - functions compile to native code
- target 64-bit x86 for heap capacity
 - ▶ space for 2³¹ (2 billion) cons cells (16 GB)
- efficient scannerless parsing + abbreviations
 - must cope with 4 gigabyte input
- graceful exits in all circumstances
 - allowed to run out of space, but must report it

Workflow

~30,000 lines of HOL4 scripts

- 1. specified input language: syntax & semantics
- 2. verified necessary algorithms, e.g.
 - compilation from source to bytecode
 - parsing and printing of s-expressions
 - copying garbage collection
- 3. proved refinements from algorithms to x86 code
- 4. plugged together to form read-eval-print loop

AST of input language

```
::= Const sexp
                                                                      ::= Val num
term
                                                              sexp
                     Var string
                                                                              Sym string
                     App func (term list)
                                                                              Dot sexp sexp
                     If term term term
                     LambdaApp (string list) term (term list)
                     Or (term list)
                     And (term list)
                                                                     (macro)
                     List (term list)
                                                                     (macro)
                     Let ((string \times term) \text{ list}) \text{ } term
                                                                     (macro)
                     LetStar ((string \times term) \text{ list}) \text{ } term
                                                                     (macro)
                      Cond ((term \times term) \text{ list})
                                                                     (macro)
                      First term | Second term | Third term
                                                                     (macro)
                      Fourth term \mid Fifth term
                                                                     (macro)
                     Define | Print | Error | Funcall
func
                     PrimitiveFun primitive | Fun string
primitive
             ::= Equal | Symbolp | SymbolLess
                     Consp | Cons | Car | Cdr |
                      Natp | Add | Sub | Less
```

compile: AST → bytecode list

bytecodePop pop one stack element PopN num pop n stack elements PushVal *num* push a constant number PushSym *string* push a constant symbol push the nth constant from system state LookupConst *num* Load *num* push the nth stack element overwrite the nth stack element Store *num* DataOp *primitive* add, subtract, car, cons, ... Jump *num* jump to program point nJumplfNil num conditionally jump to nDynamicJump jump to location given by stack top Call num static function call (faster) DynamicCall dynamic function call (slower) Return return to calling function signal a runtime error Fail Print print an object to stdout Compile compile a function definition

How do we get just-in-time compilation?

Treating code as data:

$$\forall p \ c \ q. \quad \{p\} \ c \ \{q\} \ = \ \{p * \mathsf{code} \ c\} \ \emptyset \ \{q * \mathsf{code} \ c\}$$

Definition of Hoare triple:

$$\{p\}\ c\ \{q\} = \forall s\ r.\ (p*r*\mathsf{code}\ c)\ s \Longrightarrow \exists n.\ (q*r*\mathsf{code}\ c)\ (\mathsf{run}\ n\ s)$$

I/O and efficient parsing

Jitawa implements a read-eval-print loop:

Use of external C routines adds assumptions to proof:

- reading next string from stdin
- printing null-terminated string to stdout

Read-eval-print loop

- Result of reading lazily, writing eagerly
- Eval = compile then jump-to-compiled-code
- Specification: read-eval-print until end of input

```
\frac{\text{is\_empty } (\text{get\_input } io)}{(k, io) \xrightarrow{\text{exec}} io}
```

Correctness theorem

There must be enough memory and I/O assumptions must hold.

This machine-code Hoare triple holds only for terminating executions.

```
{ init_state io* pc p* \langle terminates\_for io \rangle \}
 p: code\_for\_entire\_jitawa\_implementation \langle list of numbers
 { error_message \lor \exists io'. \langle ([], io) \xrightarrow{exec} io' \rangle * final\_state io' \}
```

Each execution is allowed to fail with an error message.

If there is no error message, then the result is described by the high-level op. semantics.

Verified code

```
$ cat verified code.s
    /* Machine code automatically extracted from a HOL4 theorem.
                                                                 */
    /* The code consists of 7423 instructions (31840 bytes).
                                                                 */
      .byte 0x48, 0x8B, 0x5F, 0x18
      .byte 0x4C, 0x8B, 0x7F, 0x10
      .byte 0x48, 0x8B, 0x47, 0x20
      .byte 0x48, 0x8B, 0x4F, 0x28
      .byte 0x48, 0x8B, 0x57, 0x08
      .byte 0x48, 0x8B, 0x37
      .byte
            0x4C, 0x8B, 0x47, 0x60
      .byte
            0x4C, 0x8B, 0x4F, 0x68
      .byte
            0x4C, 0x8B, 0x57, 0x58
            0x48, 0x01, 0xC1
      .byte
            0xC7, 0x00, 0x04, 0x4E, 0x49, 0x4C
      .byte
      .byte 0x48, 0x83, 0xC0, 0x04
            0xC7, 0x00, 0x02, 0x54, 0x06, 0x51
      .byte
            0x48, 0x83, 0xC0, 0x04
      .byte
```

Running Milawa on Jitawa

Running Milawa's 4-gigabyte booststrap process:

```
CCL 16 hours
SBCL 22 hours
Jitawa's compiler performs almost no optimisations.

Jitawa 128 hours (8x slower than CCL)
```

Parsing the 4 gigabyte input:

```
CCL 716 seconds (9x slower than Jitawa)Jitawa 79 seconds
```

Part 4:

The end-to-end result

Proving Milawa sound

semantics of Milawa's logic

inference rules of Milawa's logic

Milawa theorem prover (kernel approx. 2000 lines of Milawa Lisp)

Lisp semantics

Lisp implementation (x86) (approx. 7000 64-bit x86 instructions)

semantics of x86-64 machine

proving soundness of the source code

verification of a Lisp implementation

Jitawa
verified
LISP

Milawa



Milawa theorem prover (kernel approx. 2000 lines of Milawa Lisp)

https://raw.githubusercontent.com/HOL-Theorem-Prover/HOL/master/examples/theorem-prover/milawa-prover/core.lisp

Proving the top-level theorem

The top-level theorem:

relates the logic's semantics with the execution of the x86 machine code.

Steps:

A. formalise Milawa's logic

syntax, semantics, inference, soundness

B. prove that Milawa's kernel is faithful to the logic

- run the Lisp parser (in the logic) on Milawa's kernel
- translate (with proof) deep embedding into shallow
- prove that Milawa's (reflective) kernel is faithful to logic

C. connect the verified Lisp implementation

compose with the correctness thm for Lisp system

Theorem: Milawa is sound down to x86

There must be enough memory and input is Milawa's kernel followed by call to main for some *input*.

 $\forall input pc.$

Machine code terminates either with error message, or ...

... output lines that are all true w.r.t. the semantics of the logic.

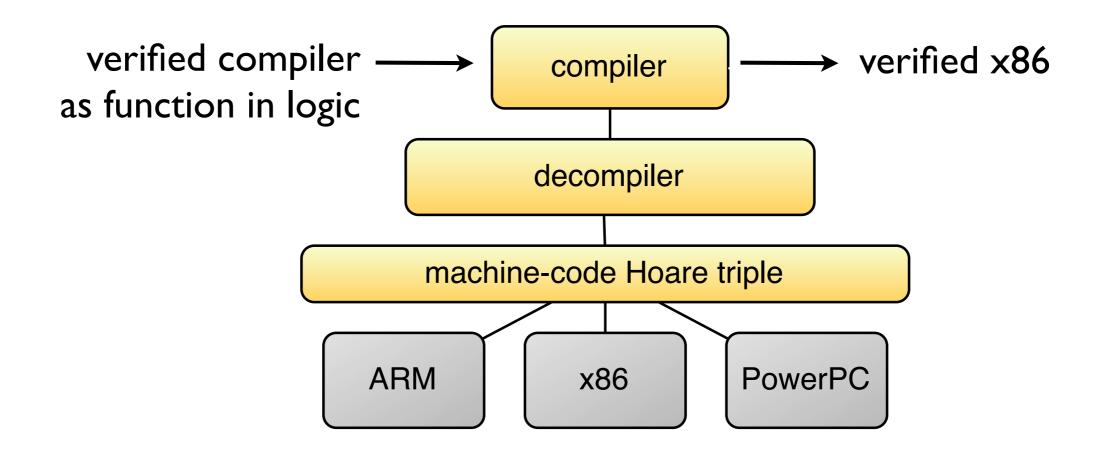
```
 | \text{line\_ok } (\pi, l) | = | (l = "NIL") \lor \\  (\exists n. \ (l = "(PRINT \ (n \dots))") \land \text{is\_number } n) \lor \\  (\exists \phi. \ (l = "(PRINT \ (THEOREM \ \phi))") \land \text{context\_ok } \pi \land \models_{\pi} \phi)
```

Final Part:

Learning from the mistakes. Doing it better.

A better compiler compiler?

The x86 for the compile function was produced as follows:



A bit cumbersome....

...should have compiled the verified compiler using itself!

Bootstrapping the compiler

Instead: we should bootstrap the verified compile function, i.e. evaluate the compiler on a deep embedding of itself within the logic:

EVAL '`compile COMPILE'`

derives a theorem:

compile COMPILE = compiler-as-machine-code



Ramana Kumar (Uni. Cambridge)



Magnus Myreen (Uni. Cambridge)



Michael Norrish (NICTA, ANU)



Scott Owens (Uni. Kent)

POPL'14

CakeML: A Verified Implementation of ML

Scott Owens ³ Michael Norrish²

Ramana Kumar* 1

Magnus O. Myreen^{† 1}

¹ Computer Laboratory, University of Cambridge, UK ² Canberra Research Lab, NICTA, Australia [‡] ³ School of Computing, University of Kent, UK

The first bootstrapping of a formally verified compiler.

CakeML, which supports a substantial subset of Standard ML. **Abstract** CakeML is implemented as an interactive read-eval-print loop (4 machine code Our correctness theorem ensures and those results permitted on the CompCert compiler for C [1, 14, 16, 29]. This interest is easy to justify: in the context of program verification, an unverified compiler forms a large and complex part of the trusted computing base. However, to our knowledge, none of the existing work on mailers for general-purpose languages has addressed all

Tomorrow at ICFP!

ICFP⁹16

A New Verified Compiler Backend for CakeML

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Michael Norrish Data61 a

Abstract

We ha end fo mediat high-le semanti 12 intermediate languages, 5 target archs, register allocation, etc. nentally compile

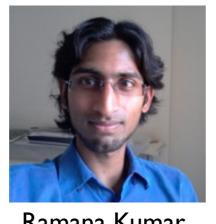
stewart et al. 2015; Ševčík et al. 2012.



Magnus Myreen



Yong Kiam Tan



Ramana Kumar



Anthony Fox



Scott Owens

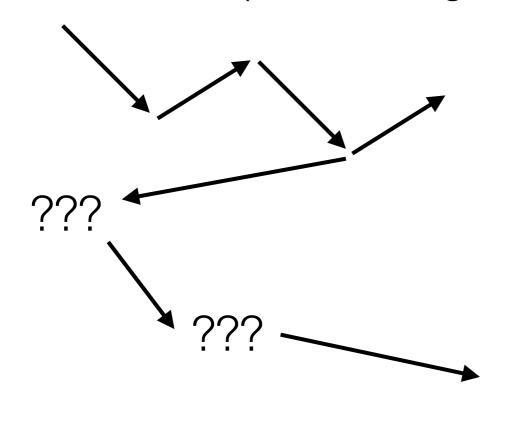


Michael Norrish

Looking back...

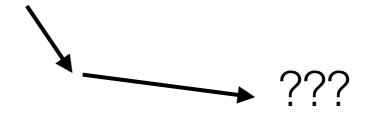
2005:

I'm a PhD student working on verification of machine code (factorial, length of a linked list)



Result:

A Verified Lisp Implementation for A Verified Theorem Prover



2005:

I'm a PhD student working on verification of machine code (factorial, length of a linked list)

basic reasoning about real machine code powerful automation verification of garbage collectors synthesis from (abstract) functional specs verified Lisp interpreters verified just-in-time compiler for Lisp Result:

A Verified Lisp Implementation for A Verified Theorem Prover

Questions?

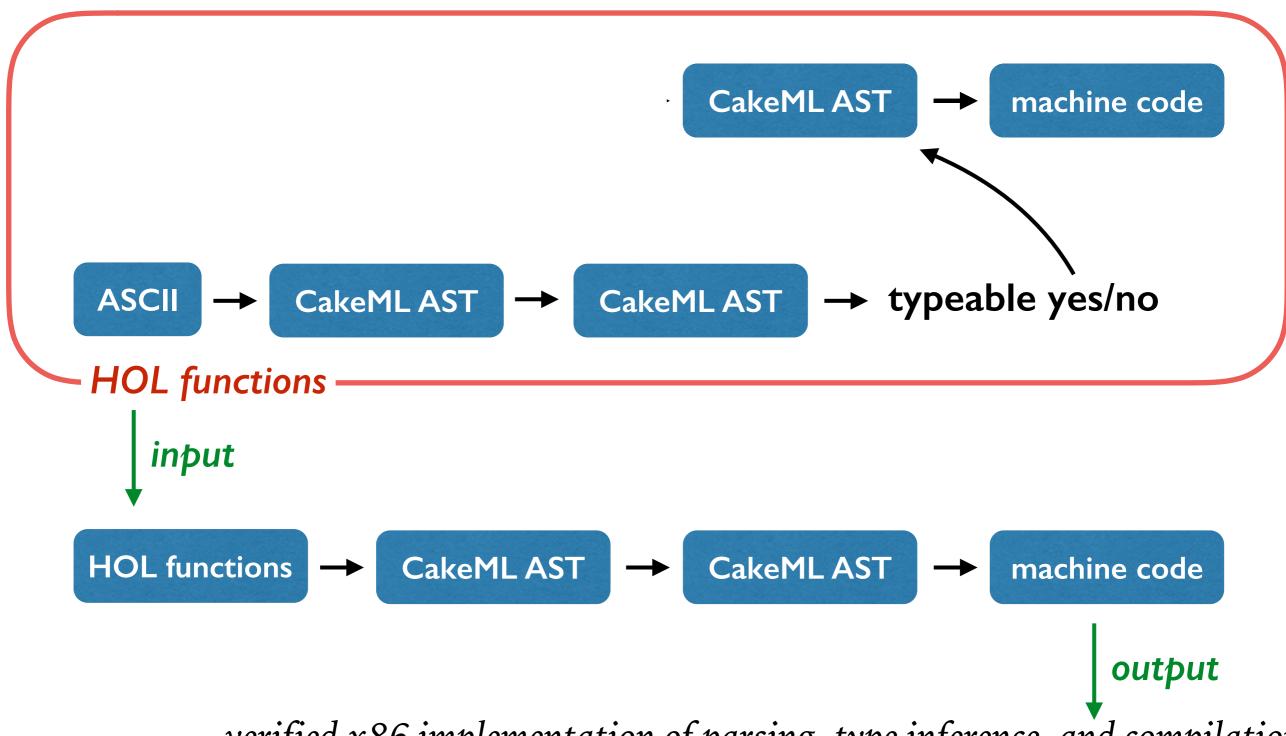
Thank you for inviting me!

verified compiler bootstrapping (ML)

Intuition for Bootstrapping

 $Proof-producing \ synthesis \qquad Verified \ compiler \ backend$ $HOL \ functions \ \rightarrow \ CakeML \ AST \ \rightarrow \ CakeML \ AST \ \rightarrow \ typeable \ yes/no$ $ASCII \ \rightarrow \ CakeML \ AST \ \rightarrow \ typeable \ yes/no$

Intuition for Bootstrapping



verified x86 implementation of parsing, type inference, and compilation